M1 常用公式

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Contents

0.1 Binomial Expansion

Formula 0.1.1.

1. $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$, *where* n *is a positive integer*

2. $C_r^n = \dfrac{n!}{(n-r)!r!}$, *where* n *and* r *are positive integers and* $r \leq n$

3. $0! = 1$ *and* $C_0^n = 1$

4. $\sum_{r=1}^{n} T_r = T_1 + T_2 + T_3 + \cdots + T_n$

5. $\sum_{r=m}^{n} k = (n-m+1)k$

6. $\sum_{r=m}^{n} ka_r = k \sum_{r=m}^{n} a_r$

7. $\sum_{r=m}^{n} a_r + \sum_{r=m}^{n} b_r = \sum_{r=m}^{n} (a_r + b_r)$

8. $(a+b)^n = a^n + C_1^n a^{n-1} b + C_2^n a^{n-2} b^2 + \cdots + C_{n-1}^n a b^{n-1} + b^n = \sum_{r=0}^{n} C_r^n a^{n-r} b^r$

0.2 Exponential Functions and Logarithmic Functions

Formula 0.2.1.

1. $e = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots + \dfrac{1}{n!} + \ldots$

2. $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \ldots$

0.3 Differentiation

Formula 0.3.1.

1. $\frac{d}{dx}(k) = 0$

2. $\frac{d}{dx}(x^n) = nx^{n-1}$, *where n is any real number*

3. $\frac{d}{dx}(ku) = k\frac{du}{dx}$

4. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

6. $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

7. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

8. $\frac{d}{dx}(e^x) = e^x$

9. $\frac{d}{dx}(a^x) = a^x \ln a$, *where* $a > 0$ *and* $a \neq 1$

10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$, *where* $x > 0$

11. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, *where* $x > 0$, $a > 0$ *and* $a \neq 1$

0.4 Applications of Differentiation

Formula 0.4.1.

1. *The equation of the tangent to the curve* $y = f(x)$ *at the point* (x, y) *is*

   $$y - y_1 = \left.\frac{dy}{dx}\right|_{x=x_1}(x - x_1)$$

2. *Increasing and decreasing functions*

   (a) *If* $f'(x) > 0$ *on* $a < x < b$ *then* $f(x)$ *is increasing on* $a \leq x \leq b$.

   (b) *If* $f'(x) < 0$ *on* $a < x < b$ *then* $f(x)$ *is decreasing on* $a \leq x \leq b$.

3. *Local extrema*

   (a) *A function* $y = f(x)$ *attains a local maximum at* $x = x_0$*,if* $f(x_0) \geq f(x)$ *for all* x *in an open interval containing* x_0. $(x_0, f(x))$ *is called a maximum point.*

   (b) *A function* $y = f(x)$ *attains a local minimum at* $x = x_0$*,if* $f(x_0) \leq f(x)$ *for all* x *in an open interval containing* x_0. $(x_0, f(x))$ *is called a minimum point.*

4. *First derivative test*

   (a) $f(x)$ *attains a local maximum at* $x = x_0$. *if* $f'(x_0) = 0$ *and* $f'(x)$ *changes from positive to negative as* x *increases through* x_0.

   (b) $f(x)$ *attains a local minimum at* $x = x_0$. *if* $f'(x_0) = 0$ *and* $f'(x)$ *changes from negative to positive as* x *increases through* x_0.

5. *Second derivative test*

   (a) $f(x)$ *attains a local maximum at* $x = x_0$ *if* $f'(x_0) = 0$ *and* $f''(x) < 0$.

   (b) $f(x)$ *attains a local minimum at* $x = x_0$ *if* $f'(x_0) = 0$ *and* $f''(x) > 0$.

6. *Concavity*

   (a) *If* $f''(x) < 0$ *on an interval. then the curve* $y = f(x)$ *is concave downwards on the interval.*

   (b) *If* $f''(x) > 0$ *on an interval. then the curve* $y = f(x)$ *is concave upwards on the interval.*

7. *Global extrema*

   (a) *A function* $y = f(x)$ *attains its global maximum at* $x = x_0$ *if* $f(x_0) \geq f(x)$ *for all* x *in the domain of* $f(x)$*, i.e. the global maximum of* $f(x)$ *is* $f(x_0)$.

   (b) *A function* $y = f(x)$ *attains its global maximum at* $x = x_0$ *if* $f(x_0) \leq f(x)$ *for all* x *in the domain of* $f(x)$*, i.e. the global maximum of* $f(x)$ *is* $f(x_0)$.

8. *Rate of change*

   For any function $u = f(t)$*, the rate of change of* u *with respect to* t *is* $\frac{du}{dt}$ *(or* $f'(t)$*).*

0.5 Indefinite Integration and its Applications

Formula 0.5.1.

1. *If* $\dfrac{d}{dx}[F(x)] = f(x)$, *then* $\int f(x)\,dx = F(x) + C$, *where* C *is an arbitrary constant.*
2. $\int k\,dx = kx + C$
3. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C,\ n \neq -1$
4. $\int \dfrac{1}{x}\,dx = \ln x|x| + C$
5. $\int e^x\,dx = e^x + C$
6. $\int e^{kx}\,dx = \dfrac{1}{k}e^{kx} + C$
7. $\int ku\,dx = k\int u\,dx$
8. $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$

0.6 Definite Integration and its Applications

Formula 0.6.1.

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b f(x)\,dx = -\int_b^a f(x)$
3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
4. $\int_a^b f(x)\,dx = \int_a^b f(u)\,du$
5. *Trapezoidal Rule*
   $\int_a^b f(x)\,dx \approx \dfrac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$, *where* $x_0 = a$, $x_n = b$, $\Delta x = \dfrac{b-a}{n}$ *and* $x_i = a + i\Delta x$ *for* $i = 0, 1, 2, 3, \ldots, n$.

0.7 Further Probability

Formula 0.7.1.

1. $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
2. $P(A \cap B) = P(A)P(B|A)$
3. $P(A|B) + P(A'|B) = 1$
4. $P(A \cap B) + P(A' \cap B) = P(B)$
5. $P(A' \cap B') = 1 - P(A \cup B)$
6. *Test for independence*
   (a) *If A and B are independent events,then $P(A \cap B) = P(A)P(B)$.*
   (b) *If $P(A \cap B) = P(A)P(B)$, then A and B are independent events.*
   (c) *If A and B are not independent events, then they are dependent events*
7. *Law of total probability*

   $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)$
8. *Bayes'theorem*

$$
\begin{aligned}
P(A_i|B) &= \frac{P(A_i \cap B)}{P(B)} \\
&= \frac{P(A_i \cap B)}{P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B)} \\
&= \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}
\end{aligned}
$$

0.8 Discrete Random Variables and Probability Distributions

Formula 0.8.1.

1. *Expectation and Variance*

   Let X be a discrete random variable with probability function $P(X = x)$.

   (a) i. *The expectation or expected value of X is given by*
   $E(X) = \mu = \sum_x xP(X = x)$

   ii. *The expectation of a function $g(X)$ of X is given by*
   $E[g(X)] = \mu = \sum_x g(X)P(X = x)$

   (b) *The variance of X is given by*
   $Var(X) = \sigma^2 = E[(X - \mu)^2] = \sum_x (x - \mu)^2 P(X = x)$
   or $Var(X) = \sigma^2 = E(X^2)] - [E(X)]^2$

   (c) *For any constants a and b,*

   i. $E(aX + b) = aE(X) + b$

   ii. $Var(aX + b) = a^2 Var(X)$

0.9 Discrete Probability Distributions

Formula 0.9.1.

1. *If X follows a Bernoulli distribution with the probability of success being p, then*

   (a) $P(X = x) = p^x(1 - p)^{1-x}$ *for* $x = 0, 1$

   (b) $E(X) = p$

   (c) $Var(X) = p(1 - p)$

2. *If $X \sim B(n, p)$, then*

   (a) $P(X = x) = C^n_x p^x(1 - p)^{n-x}$ *for* $x = 0, 1, 2, \ldots, n$

   (b) $E(X) = np$

   (c) $Var(X) = np(1 - p)$

3. *If $X \sim Po(\lambda)$, where $\lambda > 0$, then*

   (a) $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ *for* $x = 0, 1, 2, \ldots$

   (b) $E(X) = \lambda$

   (c) $Var(X) = \lambda$

0.10 Continuous Random Variables and Normal Distribution

Formula 0.10.1.

1. *If* $X \sim N(\mu, \sigma^2)$, *then*
   (a) $\dfrac{X-\mu}{\sigma} = Z \sim N(0,1)$,
   (b) $P(x_1 \leq X \leq x_2) = P(\dfrac{x_1-\mu}{\sigma}) \leq Z \leq \dfrac{x_2-\mu}{\sigma}$

0.11 Sampling Distribution and Parameter Estimation

Formula 0.11.1.

1. *Let* $\overline{X}$ *be the sample mean of a random sample of size* n *drawn from a population with a mean of* μ *and a variance of* σ^2.
   (a) *For any population,* $E(\overline{X}) = \mu$ *and* $Var(\overline{X}) = \dfrac{\sigma^2}{n}$.
   (b) *For a normal population,* $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$.
   (c) *For a non-normal population,* $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ *approximately for* $n \geq 30$.
2. *Let* $(X_1, X_2, \ldots, X_n,)$ *be a random sample of size* n *drawn from a population.*
   (a) *The unbiased estimator of the population mean* μ *is the sample mean* $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$.
   (b) *The unbiased estimator of the population variance* σ^2 *is the sample variance* $S^2 = \dfrac{\sum_{i-1}^{n}(X_i - \overline{X})^2}{n-1}$ *or* $\dfrac{\sum_{i-1}^{n} X_i^2 - n\overline{X}^2}{n-1}$